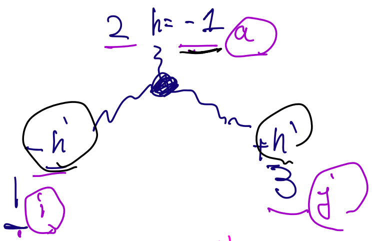
YM, Gr, general matter couplings, (elementary particles (massless))

spin-1 2

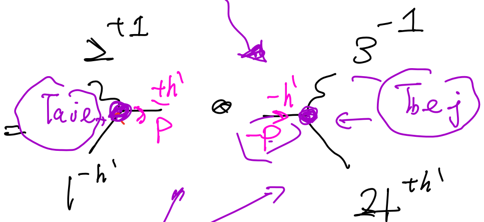


$$= \langle 12\rangle^{2h'+1} \langle 23\rangle^{1-2h'} \langle 13\rangle^{-1} (T_a)_{ij}$$

$h' \geq 0$

$$A_4(1_i^{-h'}\, 2_a^{+1}\, 3_b^{-1}\, 4_j^{+h'})\Big|_{s=0}$$

$1^{-h'}\ 2^{+2}\ 3^{-2}\ 4^{h'}$



① On the $S=0$ pole residue

$h' \leq 2.$

$$= \frac{(\langle 13\rangle [24])^{2h'}}{u} ([2|P_1 - P_4|3\rangle)^{2-2h'}$$

$[2|P_1|3\rangle - [2|P_4|3\rangle$

$$\equiv R_s\, T\, T$$

$h' \geq 0$

$\rightarrow 2-2h' < 0$

$\boxed{h' \leq 1}$

$$R_s \sim \frac{\cdots\cdots}{([2|P_1 - P_4|3\rangle)^{\alpha}}$$

not a mandelstam pole

$\tilde\lambda_2^{\dot\alpha} \lambda_3^{\alpha} = P^{\dot\alpha\alpha}$

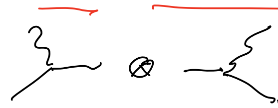
$(P_1 - P_4) \,/\!/\, P^{\dot\alpha\alpha}$

Spurious singularity

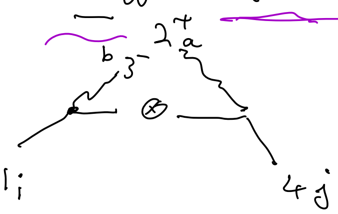
Consistent factorization for A_4 $\rightarrow$ spin-1 can only couple to $h \leq 1$

$\frac{3}{2}$ cannot be charged

$$A_4(1_i^{-h}\, 2_a^{+1}\, 3_b^{-1}\, 4_j^{+h})\Big|_{s=0} = R_s \times (T_a)_{ie} (T_b)_{ej}$$



$$A_4(\cdots)\big|_{u=0} = R_u \times (T_b)_{ie}(T_a)_{ej}$$



$$R_s = \frac{(\langle 13\rangle[24])^{2h}\,([2|P_1-P_4|3\rangle)^{2-2h}}{u} \qquad s=0 \quad u=-t$$

$$R_u = \frac{(\qquad)\,(\qquad)}{s}$$

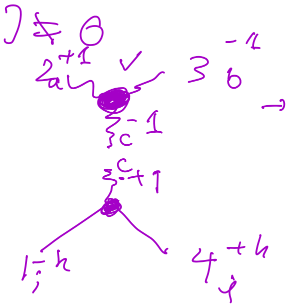
$$A_4(1_i^{-h}\,2_a^{+1}\,3_b^{-1}\,4_j^{+h}) = \boxed{\frac{(\langle 13\rangle[24])^{2h}\,([2|P_1-P_4|3\rangle)^{2-2h}}{s\,u}\,(T_a)_{ie}(T_b)_{ej}} + \text{regular}.$$

correct if $(T_a)_{ie}(T_b)_{ej} = (T_b)_{ie}(T_a)_{ej}$

$$[T_a, T_b] = 0$$

if $[T_a, T_b] \neq 0$

t-channel factorization



$\rightarrow (\qquad)\, f^{abc}$

$\rightarrow (\qquad)\, T_{c\,ij}$

$$R_t = \frac{(\langle 13\rangle[24])^{2h}\,([2|P_1-P_4|3\rangle)^{2-2h}}{s}\, f^{abc}\,T_{c\,ij}$$

4-pt amplitude is consistent only if

$$\boxed{[T_a, T_b]_{ij} = f^{abc}\,T_{c\,ij}}$$

$$A_4(1_i^{-h}\, 2_a^{+1}\, 3_b^{-1}\, 4_j^{+h})\Big|_{t=0} = \left(\langle 13\rangle[24]\right)^{2h}\left([2|(p_1-p_4)|3\rangle\right)^{2-2h}$$

$$\times\left(\frac{(T_a T_b)_{ij}}{s\,\boxed{t}} + \frac{(T_b T_a)_{ij}}{\boxed{t}\,u}\right)$$

$t=0 \quad s=-u \qquad \dfrac{T_a T_b}{s} + \dfrac{T_b T_a}{u}$

$$= \frac{1}{s}\left(T_a T_b - T_b T_a\right)$$

$$= \frac{1}{s} f^{abc} T_c$$

Massive scattering:

$$A_n(\langle ij\rangle, [ij]) \to A_n(\{\lambda_i, \tilde\lambda_i\}_{massless}, \{\tilde\lambda_{iI}, \lambda_i^I\}_{massive})$$

SU(2) little group

$$\to A_n(\{\lambda_i, \tilde\lambda_i\}_{massless}, \{\lambda_j^I\}_{massive})$$

① $\quad p_i^{\alpha\dot\alpha}\tilde\lambda_{i\dot\alpha I} = m\,\lambda_{iI}^{\alpha}$

② $\quad$ SU(2) irrep $\quad \odot^{\{I_1 I_2 \cdots I_{2s}\}}$

spin-s

$s=\frac{1}{2} \qquad \odot^{I_1}$

$s=1 \qquad \odot^{I_1 I_2} \leftarrow$ symmetric $\qquad \left.\begin{matrix}\odot^{11}\\ \odot^{12}\\ \odot^{22}\end{matrix}\right\}\ 3$

$s=2 \qquad \odot^{I_1 I_2 I_3 I_4} \qquad \left\{\begin{matrix}\odot^{1111}\\ \odot^{1112}\\ \odot^{1122}\\ \vdots\end{matrix}\right\}\ 5$

$$\begin{bmatrix} \Theta^{\cdots} & - \\ \vdots & \\ \Theta^{2222} & \end{bmatrix}$$

$\boxed{2S+1}$

$$S = \mathbb{1} + i\,\underline{T}$$

$$S^{\dagger} S = \underline{\mathbb{1}} = (\mathbb{1} - i T^{\dagger})(\mathbb{1} + iT) = \mathbb{1}$$

$$\boxed{\mathrm{Im}(T) = T T^{\dagger}}$$

$$T = \lambda T' + \lambda^{2} \cdots$$

$$\mathrm{Im}(\underline{A_4}) = T(A_3)\, T(A_3)$$

$S + i\epsilon$

Massive states

$$A_n\left(\{\lambda_i \tilde{\lambda}_i\}_{massless}, \{\lambda_i^{\alpha I}\}_{massive}\right)$$

n = 3

$$A_3(1^{s_1} 2^{s_2} 3^{s_3}) = \mathcal{A}_3 \overbrace{\{I_1 I_2 \cdots I_{2s_1}\}}^{\lambda_1} \overbrace{\{J_1 J_2 \cdots J_{2s_2}\}}^{\lambda_2} \overbrace{\{K_1 K_2 \cdots K_{2s_3}\}}^{\lambda_3}$$

irrep s_1 under SU(2)

I rank $2s_1$ in SU(2) indices

$$= \left(\lambda_{1\alpha_1}^{I_1} \lambda_{1\alpha_2}^{I_2} \cdots \lambda_{1\cdot\alpha_{2s_1}}^{I_{2s_1}}\right)\left(\lambda_{2\beta_1}^{J_1} \lambda_{2\beta_2}^{J_2} \cdots \lambda_{2\beta_{2s_2}}^{J_{2s_2}}\right)\left(\lambda_{3\gamma_1}^{K_1} \cdots \lambda_{3\gamma_{2s_3}}^{K_{2s_3}}\right) \mathcal{O}^{\{\alpha_1 \cdots \alpha_{2s_1}\}\{\beta_1 \cdots \beta_{2s_2}\}\{\gamma_1 \cdots \gamma_{2s_3}\}}$$

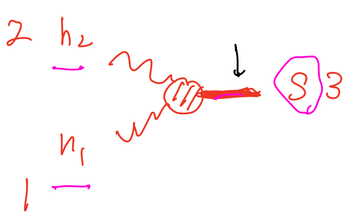
$\epsilon_{\alpha_1\alpha_2 \cdots \alpha_{2s_2}}$

non-trivial

Indices in the same space (SL(2,C) Lorentz group)

2 2-component vectors to span space

Exp1:



$$A_3(1^{h_1} 2^{h_2} 3^{S}) = \lambda_{3\alpha_1}^{\{I_1} \lambda_{3\alpha_2}^{I_2} \cdots \lambda_{3\alpha_{2S}}^{I_{2S}\}} \mathcal{O}^{\{\alpha_1\alpha_2 \cdots \alpha_{2S}\}}$$

u v

2 2-component $(\lambda_1^\alpha, \lambda_2^\alpha)$

$\mathcal{O}^{\{\alpha_1\alpha_2 \cdots \alpha_{2S}\}}$

(a) must have 2S spinor indices $a+b=2S$

(b) must have h_1 little group weight for leg 1

(c) - - h_2 - - ~ h_2

$$(u)^a (v)^b [12]^c \left(\tilde{\lambda}_1^\alpha \tilde{\lambda}_2^\beta \epsilon_{\alpha\beta}\right)$$

$$-\frac{a}{2} + \frac{c}{2} = h_1$$

$$-\frac{b}{2} + \frac{c}{2} = h_2$$

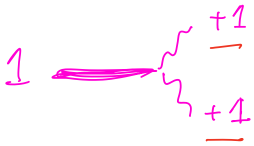
$$\boxed{\begin{aligned} c &= h_1 + h_2 + S \\ a &= -h_1 + h_2 + S \\ b &= h_1 - h_2 + S \end{aligned}}$$

$S = 1 \quad h_1 = +1 \quad h_2 = +1$

$a = 1$ ✓

$b = 1$ ✓

$c = 3$



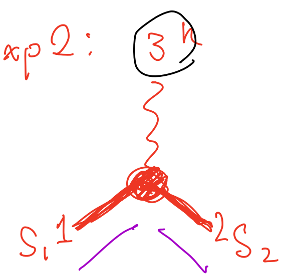
$$\bigcirc^{\alpha_1\alpha_2} = \boxed{g\, \lambda_1^{\{\alpha_1} \lambda_2^{\alpha_2\}} [12]^3}$$

$1 \leftrightarrow 2$ inconsistent $\leftrightarrow$ bose symmetric

anti-sy $1 \leftrightarrow 2$.

Yang's Theorem (a massive spin-1 cannot decay in to identical vectors)

Exp 2:



$$A_3(1^{S_1}\, 2^{S_2}\, 3^h) \begin{cases} \to m_1 = m_2 = m \quad (1) \\ \to m_1 \neq m_2 \quad \cdots (2) \end{cases}$$

Trivial

equal mass:

$$A_3(1^{S_1}\, 2^{S_2}\, 3^h) = \begin{pmatrix} \lambda_{1\alpha_1}^{\{I_1} & \lambda_{1\alpha_2}^{I_2} & \cdots & \lambda_{1\alpha_{2S_1}}^{I_{2S_1}\}} \\ \lambda_{2\beta_1}^{\{J_1} & \lambda_{2\beta_2}^{\cdots} & \cdots & \lambda_{2\beta_{2S_2}}^{J_{2S_2}\}} \end{pmatrix}$$

2-vector $\{ \lambda_3^\alpha,\ \epsilon^{\alpha\beta}\lambda_{3\beta} \}$

$$\bigcirc^{\{\alpha_1 \cdots \alpha_{2S_1}\}\{\beta_1 \cdots \beta_{2S_1}\}}$$

$$\boxed{f(\lambda_3^\alpha, \epsilon_{\alpha\beta}, x)}$$

$\bigcirc$ satisfy $\begin{cases} \{\alpha_1 \cdots \alpha_{2S_1}\} \ \{\beta_1 \cdots \beta_{2S_2}\} \\ \text{helicity weight } h \end{cases}$

$$P_1 + P_3 = -P_2$$

$$(P_1 + P_3)^2 = P_2^2 = \cancel{m^2}$$

$$\| $$

$$P_1^2 + 2P_1 \cdot P_3 \quad \Rightarrow \quad 2P_1 \cdot P_3 = 0$$

$$\| \qquad\qquad\qquad \|$$

$$\cancel{m^2}$$

For equal mass

$$\boxed{x\,\underset{-\frac{1}{2}}{\lambda_3^{\alpha}} = \frac{p_1^{\alpha}{}_{\dot\alpha}\,\underset{+\frac{1}{2}}{\tilde\lambda_3^{\dot\alpha}}}{m}}$$

+1

defined kinematically

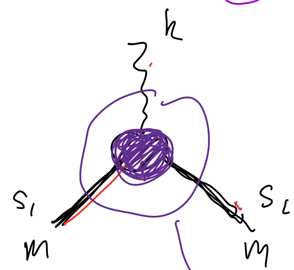
$$\Rightarrow \frac{p_2^{\alpha}{}_{\dot\alpha}\tilde\lambda_3^{\dot\alpha}}{m} = -\frac{p_1^{\alpha}{}_{\dot\alpha}\tilde\lambda_3^{\dot\alpha}}{m}$$

$$\lambda_3^{\alpha}\, p_{1\alpha\dot\alpha}\tilde\lambda_3^{\dot\alpha} = 0$$

$$\lambda_3^{\alpha}\, p_1^{\beta}{}_{\dot\alpha}\tilde\lambda_3^{\dot\alpha}\,\epsilon_{\alpha\beta} = 0$$

$$\lambda_3^{\alpha}\lambda_3^{\beta}\,\epsilon_{\alpha\beta} = 0$$

Summary



$$\Rightarrow \mathcal{O}^{\{\alpha_1\cdots\alpha_{2s_1}\}\{\beta_1\cdots\beta_{2s_2}\}}$$

$$\left(\underset{-\frac{1}{2}}{\lambda_3^{\alpha}},\ \epsilon_{\alpha\beta},\ \underset{+1}{x}\right)$$

$h = +1 \quad +2$

$s_1 = s_2 = s$

① $s = \frac{1}{2}$

$$\mathcal{O}^{\alpha_1\beta_1} = x\,\epsilon^{\alpha_1\beta_1} + x^2\lambda_3^{\alpha_1}\lambda_3^{\beta_1}$$

$$= m x^2\left(\epsilon^{\alpha_1\beta_1} + g\,x\,\frac{\lambda_3^{\alpha_1}\lambda_3^{\beta_1}}{m}\right)$$

$g=2$ classical

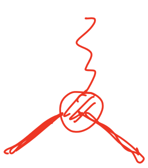
Minimal Coupling

$(g-2)$ factor

$$A_3 = \frac{\lambda_{1\alpha_1}^{I_1}\lambda_{2\beta_1}^{I_2}}{m}\,\mathcal{O}^{\alpha_1\beta_1} = \boxed{\langle 1^{I_1} 2^{I_2}\rangle\, x}$$

$$\bar\Psi \not{A} \Psi$$
$$\Downarrow$$
$$\bar\Psi \not{\epsilon} \Psi$$

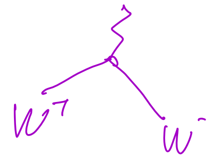
$S = 1$



$$\mathcal{O}^{\{\alpha_1\alpha_2\}\{\beta_1\beta_2\}} = \boxed{x^2\left(\epsilon^{\alpha_1\beta_1}\epsilon^{\alpha_2\beta_2}\right.} + g_1\,\epsilon^{\alpha_1\beta_1}\lambda_3^{\alpha_2}\lambda_3^{\beta_2}\,x/m + g_2\,\lambda_3^{\alpha_1}\lambda_3^{\alpha_2}\lambda_3^{\beta_1}\lambda_3^{\beta_2}\,x^2\Big)$$

$g-2$

Minimal coupling



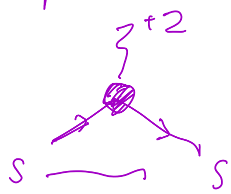
~~anomalous quadrupole moment~~ m^2

$S = 26$ $\quad = x_c \big(\overbrace{\epsilon \cdots \epsilon}^{13} + \cdots \big)$

$(2S+1)$ - possible couplings

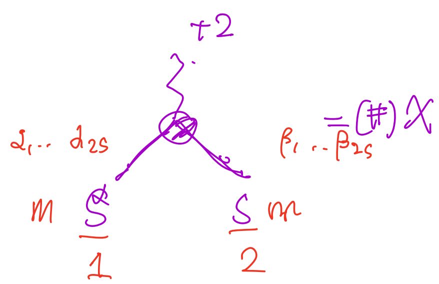
Derivation of spin-statistic theorem

1. spin-s state, equivalence principle tells us that it must couple to gravity



must exist.

2. From the discussion above. (Lorentz inv, irreps of $SU(2)$, (x_i))
Little group



$1 \leftrightarrow 2$

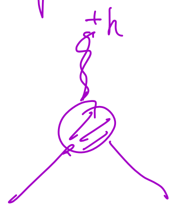
$\alpha \leftrightarrow \beta$

$$(-) \quad (-) \quad (-)$$

$$\Big[\epsilon^{\alpha_1\beta_1} \epsilon^{\alpha_2\beta_2} \cdots \epsilon^{\alpha_{2S}\beta_{2S}} \quad (-)^{2S}$$
$$+ \epsilon^{\alpha_1\beta_1} \cdots \epsilon^{\alpha_{2S-1}\beta_{2S-1}} \lambda_3^{\alpha_{2S}} \lambda_3^{\beta_{2S}} \chi$$
$$+ \epsilon\,\epsilon\,\epsilon \cdots \lambda_3 \lambda_3 \lambda_3 \lambda_3 \chi^2$$
$$+ \quad \vdots$$
$$+ \underbrace{\lambda_3 \cdots \lambda_3}_{2S} \Big]$$

$$\mathcal{O}^{\{\alpha_1\cdots\alpha_{2s}\}\{\beta_1\cdots\beta_{2s}\}}\Big|_{1\leftrightarrow 2} = (-)^{2s}\,\mathcal{O}^{\{\ldots\}\{\ldots\}}$$

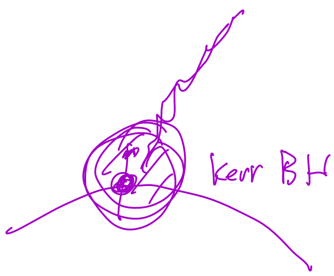
For any spin-s we always have minimal coupling



$= \boxed{x^h\left(\epsilon^{\alpha_1\beta_1}\epsilon\cdots\epsilon^{\alpha_{2s}\beta_{2s}}\right)}$ For any s

analy continue to $s\to\infty$

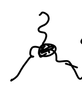
$s\to\infty$ $\hbar\to 0$ $s\hbar = S$ fix.



Kerr BH

$|s_1' s_2'\rangle = S\,|s_1 s_2\rangle$

$\int\sqrt{g}R + R^3 \Rightarrow$



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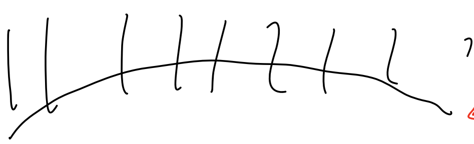

?

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